A longitudinal component in massive gravitational waves arising from a bimetric theory of gravity

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November 6, 2008

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Abstract

After a brief review of the work of de Paula, Miranda and Marinho on massive gravitational waves arising from a bimetric theory of gravity, in this paper it is shown that the presence of the mass generates a longitudinal component in a particular polarization of the wave. The effect of this polarization on test masses is performed using the geodesic deviation. A the end of this paper the detectability of this particular polarization is also discussed, showing that its angular dependence could, in principle, discriminate such polarization with respect the two ones of general relativity, if present or future detectors will achieve a high sensitivity.

1   Introduction

The design and construction of a number of sensitive detectors for gravitational waves (GWs) is underway today. There are some laser interferometers like the VIRGO detector, being built in Cascina, near Pisa by a joint Italian-French collaboration [1, 2], the GEO 600 detector, being built in Hannover, Germany by a joint Anglo-Germany collaboration [3, 4], the two LIGO detectors, being built in the United States (one in Hanford, Washington and the other in Livingston, Louisiana) by a joint Caltech-Mit collaboration [5, 6], and the TAMA 300 detector, being built near Tokyo, Japan [7, 8]. There are many bar detectors currently in operation too, and several interferometers and bars are in a phase of planning and proposal stages.

The results of these detectors will have a fundamental impact on astrophysics and gravitation physics. There will be lots of experimental data to be analyzed, and theorists will be forced to interact with lots of experiments and data analysts to extract the physics from the data stream.

Detectors for GWs will also be important to confirm or ruling out the physical consistency of general relativity or of any other theory of gravitation [9, 10, 11, 12]. This is because, in the context of extended theories of gravity, some differences from general relativity and the others theories can be seen starting by the linearized theory of gravity [9, 10, 12].

Some papers in the literature have also shown that a massive component of gravitational waves could in principle be present in alternative theories of gravity, like scalar-tensor theories [12, 13, 14], high-order theories [15] and a bimetric theory [16].

Focusing our attention on this bimetric theory, after a brief review of the work of de Paula, Miranda and Marinho on massive gravitational waves from such as theory [16], which is due to provide a context to bring out the relevance of the results, in this paper it is shown that the presence of the mass generates a longitudinal component in a particular polarization of the wave. The effect of this polarization on test masses is performed using the geodesic deviation. A the end of this paper the detectability of this particular polarization is also discussed, showing that its angular dependence could, in principle, discriminate such polarization with respect the two ones of general relativity, if present or future detectors will achieve a high sensitivity.

2 A review of massive gravitational waves from the bimetric theory

An extension of linearized general relativity which takes into account massive gravitons gives a weak-field stress-energy tensor [16]

$$T_{\mu\nu}^{(m)} = -\frac{m_g}{8\pi}\{h_{\mu\nu} - \frac{1}{2}[(g_0^{-1})^{\alpha\beta}h_{\alpha\beta}](g_0)_{\mu\nu}\}, \tag{1}$$

where m_g is the mass of the graviton, and $(g_0)_{\mu\nu}$ the non-dynamical background metric (note: differently from [16] in this paper we work with $G = 1$, $c = 1$ and $\hbar = 1$). In this way the field equations can be obtained in an einstenian form like

$$G_{\mu\nu} = -8\pi(T_{\mu\nu} + T_{\mu\nu}^{(m)}), \tag{2}$$

where $T_{\mu\nu}$ is the ordinary stress-energy tensor of the matter. General relativity is recovered in the limit $m_g \to 0$.

Calling $g_{\mu\nu}$ the dynamic metric and putting

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \tag{3}$$

with $|h_{\mu\nu}| \ll 1$ equation (2) can be linearized in vacuum (i.e. $T_{\mu\nu} = 0$) obtaining

$$[]\overline{h}_{\mu\nu} = m^2 \overline{h}_{\mu\nu}, \tag{4}$$

where $[]$ is the d'Alembertian operator and $\overline{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{h}{2}\eta_{\mu\nu}$.
The general solution of this equation is [16]

$$\overline{h}_{\mu\nu} = e_{\mu\nu} \exp(ik^\alpha x_\alpha), \tag{5}$$

where $e_{\mu\nu}$ is the polarization tensor.
The condition of normalization $k^\alpha k_\alpha = m^2$ gives $k = \sqrt{\omega^2 - m^2}$ and a speed of propagation

$$v(\omega) = \frac{\sqrt{\omega^2 - m^2}}{\omega}, \tag{6}$$

which is exactly the velocity of a massive particle with mass m (it is also the group-velocity of a wave-packet [12, 14, 15]).

Thus, assuming that the wave is propagating in the z direction, the metric perturbation (5) can be rewritten like

$$\overline{h}_{\mu\nu} = e_{\mu\nu} \exp(ikz - i\omega). \tag{7}$$

Using a tetrade formalism, the authors of [16] found six independent polarizations states (see equations 28-33 of [16]). In the following section it will be shown that, from the polarization labelled with Φ_{22} in [16] (equations 32 and 38 of [16]), a longitudinal force is present.

3   The origin of a longitudinal component

Let us consider equation 38 of [16]. Putting $h_g \equiv h_{00} + h_{33}$, this equation can be rewritten as

$$\Phi_{22} = \frac{1}{8}h_g(t, z). \tag{8}$$

Taken in to account only the Φ_{22} polarization in equation (5) one gets

$$\overline{h}_{\mu\nu}(t, z) = \frac{1}{8}h_g(t, z)\eta_{\mu\nu} \tag{9}$$

and the corrispondent line element is the conformally flat one

$$ds^2 = [1 + \frac{1}{8}h_g(t, z)](-dt^2 + dz^2 + dx^2 + dy^2). \tag{10}$$

Because the analysis on the motion of test masses is performed in a laboratory environment on Earth, the coordinate system in which the space-time is locally flat is typically used and the distance between any two points is given simply

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by the difference in their coordinates in the sense of Newtonian physics [12, 13, 14, 15, 16, 17, 18]. This frame is the proper reference frame of a local observer, located for example in the position of the beam splitter of an interferometer. In this frame gravitational waves manifest themself by exerting tidal forces on the masses (the mirror and the beam-splitter in the case of an interferometer). A detailed analysis of the frame of the local observer is given in ref. [17], sect. 13.6. Here only the more important features of this coordinate system are recalled:

the time coordinate x_0 is the proper time of the observer O;

spatial axes are centered in O;

in the special case of zero acceleration and zero rotation the spatial coordinates x_j are the proper distances along the axes and the frame of the local observer reduces to a local Lorentz frame: in this case the line element reads [17]

$$ds^2 = -(dx^0)^2 + \delta_{ij}dx^i dx^j + O(|x^j|^2)dx^\alpha dx^\beta. \tag{11}$$

The effect of the gravitational wave on test masses is described by the equation

$$\ddot{x}^i = -\widetilde{R}^i_{0k0}x^k, \tag{12}$$

which is the equation for geodesic deviation in this frame.

Thus, to study the effect of the massive gravitational wave on test masses, $\widetilde{R}^i_{0k0}$ has to be computed in the proper reference frame of the local observer. But, because the linearized Riemann tensor $\widetilde{R}_{\mu\nu\rho\sigma}$ is invariant under gauge transformations [12, 15, 17], it can be directly computed from eq. (9).

From [17] it is:

$$\widetilde{R}_{\mu\nu\rho\sigma} = \frac{1}{2}\{\partial_\mu\partial_\beta h_{\alpha\nu} + \partial_\nu\partial_\alpha h_{\mu\beta} - \partial_\alpha\partial_\beta h_{\mu\nu} - \partial_\mu\partial_\nu h_{\alpha\beta}\}, \tag{13}$$

that, in the case eq. (9), begins

$$\widetilde{R}^\alpha_{0\gamma 0} = \frac{1}{16}\{\partial^\alpha\partial_0 h_g \eta_{0\gamma} + \partial_0\partial_\gamma h_g \delta^\alpha_0 - \partial^\alpha\partial_\gamma h_g \eta_{00} - \partial_0\partial_0 h_g \delta^\alpha_\gamma\}; \tag{14}$$

the different elements are (only the non zero ones will be written):

$$\partial^\alpha\partial_0 h_g \eta_{0\gamma} = \left\{ \begin{array}{ll} \partial_t^2 h_g & for \quad \alpha = \gamma = 0 \\ -\partial_z\partial_t h_g & for \quad \alpha = 3; \gamma = 0 \end{array} \right\} \tag{15}$$

$$\partial_0\partial_\gamma h_g \delta^\alpha_0 = \left\{ \begin{array}{ll} \partial_t^2 h_g & for \quad \alpha = \gamma = 0 \\ \partial_t\partial_z h_g & for \quad \alpha = 0; \gamma = 3 \end{array} \right\} \tag{16}$$

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$$- \partial^\alpha \partial_\gamma h_g \eta_{00} = \partial^\alpha \partial_\gamma h_g = \begin{cases} -\partial_t^2 h_g & for \quad \alpha = \gamma = 0 \\ \partial_z^2 h_g & for \quad \alpha = \gamma = 3 \\ -\partial_t \partial_z h_g & for \quad \alpha = 0; \gamma = 3 \\ \partial_z \partial_t h_g & for \quad \alpha = 3; \gamma = 0 \end{cases} \tag{17}$$

$$- \partial_0 \partial_0 h_g \delta^\alpha_\gamma = -\partial_z^2 h_g \quad for \quad \alpha = \gamma \; . \tag{18}$$

Now, putting these results in eq. (14) one obtains:

$$\widetilde{R}^1_{010} = -\tfrac{1}{16} \ddot{h}_g$$

$$\widetilde{R}^2_{010} = -\tfrac{1}{16} \ddot{h}_g \tag{19}$$

$$\widetilde{R}^3_{030} = \tfrac{1}{16} [] h_g.$$

But, putting the field equation (4) in the third of eqs. (19) it is

$$\widetilde{R}^3_{030} = \frac{1}{16} m^2 h_g, \tag{20}$$

which shows that the field is not transversal.
Infact, using eq. (12) it results

$$\ddot{x} = \frac{1}{16} \ddot{h}_g x, \tag{21}$$

$$\ddot{y} = \frac{1}{16} \ddot{h}_g y \tag{22}$$

and

$$\ddot{z} = -\frac{1}{16} m^2 h_g(t, z) z. \tag{23}$$

Then the effect of the mass is the generation of a *longitudinal* force (in addition to the transverse one).

4   Geodesic deviation

For a better understanding of this longitudinal force, let us analyse the effect on test masses in the context of the geodesic deviation.

Following [14] one puts

$$\widetilde{R}^i_{0j0} = \frac{1}{16} \begin{pmatrix} -\partial_t^2 & 0 & 0 \\ 0 & -\partial_t^2 & 0 \\ 0 & 0 & m^2 \end{pmatrix} h_g(t, z) = -\frac{1}{16} T_{ij} \partial_t^2 h_g + \frac{1}{16} L_{ij} m^2 h_g. \tag{24}$$

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Here the transverse projector with respect to the direction of propagation of the GW $\widehat{n}$, defined by

$$T_{ij} = \delta_{ij} - \widehat{n}_i \widehat{n}_j, \tag{25}$$

and the longitudinal projector defined by

$$L_{ij} = \widehat{n}_i \widehat{n}_j \tag{26}$$

have been used. In this way the geodesic deviation equation (12) can be rewritten like

$$\frac{d^2}{dt^2} x_i = \frac{1}{16} \partial_t^2 h_g T_{ij} x_j - \frac{1}{16} m^2 h_g L_{ij} x_j. \tag{27}$$

Thus it appears clear what was claimed in previous section: the effect of the mass present in the GW generates a longitudinal force proportional to m^2 which is in addition to the transverse one. But if $v(\omega) \to 1$ in eq. (6) we get $m \to 0$, and the longitudinal force vanishes. Thus it is clear that the longitudinal mode arises from the fact that the GW does no propagate at the speed of light.

5 Detectability of the polarization and angular dependence

Now, let us analize the dectability of the polarization (8) computing the pattern function of a detector to this massive component. One has to recall that it is possible to associate to a detector a *detector tensor* that, for an interferometer with arms along the $\hat{u}$ e $\hat{v}$ directions with respect the propagating gravitational wave (see figure 1), is defined by [2, 13, 14]

$$D^{ij} \equiv \frac{1}{2} (\hat{v}^i \hat{v}^j - \hat{u}^i \hat{u}^j). \tag{28}$$

If the detector is an interferometer [1, 2, 3, 4, 5, 6, 7, 8, 21], the signal induced by a gravitational wave of a generic polarization, here labelled with $s(t)$, is the phase shift, which is proportional to [2, 21]

$$s(t) \sim D^{ij} \widetilde{R}_{i0j0} \tag{29}$$

and, using equations (24), one gets

$$s(t) \sim - \sin^2 \theta \cos 2\phi. \tag{30}$$

The angular dependence (30), which is shown in figure 2, is different from the two well known standard ones arising from general relativity which are, respectively $(1 + \cos^2 \theta) \cos 2\phi$ for the $+$ polarization and $- \cos \theta \sin 2\vartheta$ for the $\times$ polarization. Thus, in principle, the angular dependence (30) could be used to discriminate among the bimetric theory and general relativity, if present or future detectors will achieve a high sensitivity.
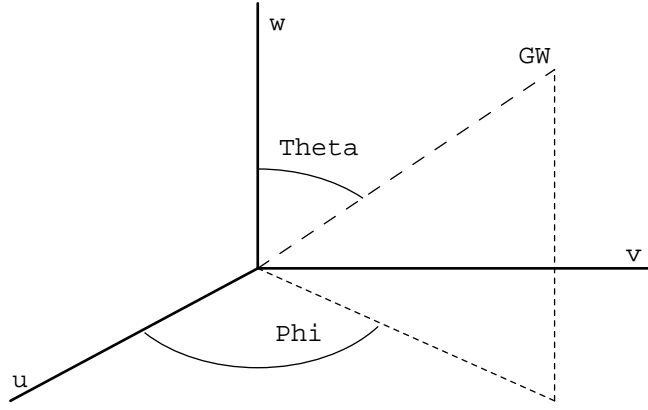
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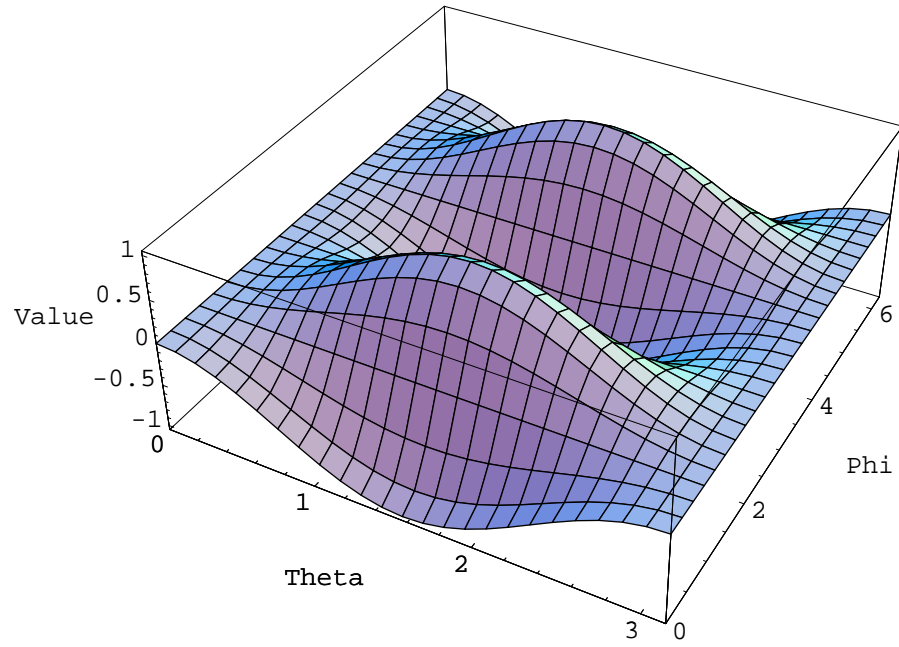Figure 1: a GW propagating from an arbitrary direction

Figure 2: the angular dependence (30)

6   Conclusions

After a brief review of the work of de Paula, Miranda and Marinho on massive gravitational waves arising from a bimetric theory of gravity, in this paper it has been shown that the presence of the mass generates a longitudinal component in a particular polarization of the wave. The effect of this polarization on test masses has been performed using the geodesic deviation. A the end of this paper the detectability of this particular polarization has also been discussed, showing that its angular dependence could, in principle, discriminate such polarization with respect the two ones of general relativity if present or future detectors will achieve a high sensitivity.

As a final remark, it seems from the analysis in the last section of this paper, that the investigation of massive component of gravitational waves could constitute a further tool to discriminate among several relativistic theories of gravity on the ground [12, 15, 19, 20].

Acknowledgements

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